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THERMAL RADIATION BETWEEN PARALLEL PLATES SEPARATED BY AN ABSORBING–EMITTING NONISOTHERMAL GAS

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Abstract—A complete formulation, including the local details of gas absorption–emission processes, has been made for thermal radiation in a parallel plate enclosure. The temperature is permitted to vary continuously between the plates, and the emissive power of the gas may have an arbitrary dependence on temperature. Thermal conductivity effects have been omitted. Solutions of the governing integral equations have been carried out for values of the single governing parameter kL (k = absorption coefficient, L = spacing) in the range 0·1 to 2·0. Temperature distributions and heat transfer results are given. For moderate values of kL, the temperature is quite uniform across the gas.

Résumé—Une formulation complète comprenant les caractéristiques détaillées et locales d'absorption et d'émission d'un gaz a été établie pour le rayonnement dans un enceint formée par façon continue entre les plaques et le pouvoir émissif du gaz peut dépendre d'une façon quelconque de la température. Les effets de conductibilité thermique ont été négligés. Des solutions aux équations intégrales du problème ont été calculées pour des valeurs du paramètre déterminant, kL, comprises entre 0,1 et 2 (k coefficient d'absorption, L écartement des plaques). Les distributions de températures et des résultats sur le le transfert de chaleur sont donnés. Pour des valeurs modérées de kL la température est pratiquement uniforme dans le gaz.

Zusammenfassung—Für die thermische Strahlung in einem Hohlraum mit parallelen Wänden wird die vollständige Formulierung angegeben einschließlich der örtlichen Einzelheiten der Absorption und Emission des Gases. Die Temperatur soll sich innerhalb der Wände kontinuierlich ändern und das Emissionsvermögen des Gases soll beliebig von der Temperatur abhängen. Wärmeleitung wird nicht in Betracht gezogen. Die maßgebenden Integralgleichungen wurden für einen Bereich des einzigen maßgebenden Parameters kL = 0,1 bis 2,0 gelöst (k = Absorptionskoeffizient, L = Abstand). Temperaturverteilungen und Wärmeübertragung sind angegeben. Für mittlere Werte kL ist die Temperatur einheitlich innerhalb des Gases.

Abstract—В статье дано решение задачи лучистого теплообмена между двумя параллельными пластинами с учётом излучения и поглощения газа, находящегося между ними. При этом предполагается непрерывное изменение температуры между пластинами, а лучеиспускательная способность газа является любой функцией температуры. Решения основных интегральных уравнений даны для значения параметра kL от 0·1 до 2·0, где k—коэффициент лучепоглощения, L—линейный размер системы. Приведены результаты расчётов температурного поля и потоков тепла. Для других значений kL температура газа принимается постоянной в плоскости сечения системы.

NOMENCLATURE

e_b = emissive power of black wall, σT^4;
e_g = emissive power of gas element, see equation (2);
k = absorption coefficient (logarithmic decrement of radiation);
L = spacing between plates;
q = net rate of heat transfer per unit area to wall;
r = distance between emitting and receiving elements;
S = internal heat source per unit volume;

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T = absolute temperature;
X = dimensionless normal co-ordinate, x/L; X_0, dimensionless co-ordinate of absorbing volume, x_0/L;
x = distance measured normal to lower plate; x_0, distance of absorbing volume from lower plate;
y = radial co-ordinate in plane of walls;
δ = infinitesimal diameter of absorbing sphere;
θ = angle between surface normal and direction of absorbing volume (see Fig. 3);
λ = integration variable;
Φ = dimensionless emissive power $(e_g - e_b)/(S/2k)$;
ϕ = dimensionless emissive power $(e_g - e_{b1})/(e_{b2} - e_{b1})$.

Differentials
dA_s = wall surface area;
dA_τ = surface area of radiating gas strip (see Fig. 2);
$d\omega$ = solid angle;
$d\tau$ = gas volume;

Subscripts
1 = lower wall;
2 = upper wall.

INTRODUCTION

THE high temperature levels achieved in modern propulsion systems (as well as in furnaces) demand that heat transfer calculations include radiation effects of absorbing–emitting gases which may lie between heat transfer surfaces. In general, the gas would be non-uniform in temperature and may contain heat sources. The general problem would involve heat transfer by convection and conduction as well as by radiation. However, it appears that even the simpler situation of purely radiant interchange between surfaces separated by a non-isothermal, absorbing–emitting gas has not yet been fully solved. It is to this latter problem to which we turn our attention here.

Specifically, consideration is given to a system composed of two parallel plates, each infinite in extent, which are separated by a gap of thickness L. The configuration is shown schematically in Fig. 1. The plate temperatures T_1 and T_2 are specially uniform, but may differ from one another. The gas filling the gap will be permitted to take a full part in the radiative heat exchange, absorbing energy and also re-emitting. The gas temperature will, in general, vary across the gap. Further, there may be a heat source in the gap.

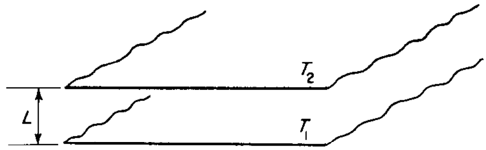

FIG. 1. Parallel plate enclosure.

Within the framework of the limiting assumptions, our aim is to make a complete formulation of the problem in which the details of local absorption and emission processes are accounted for. Applying conservation of energy to an infinitesimal gas volume element, we are led to an integral equation for the emissive power (temperature). Solutions have been carried out for several values of the governing parameter kL (k = absorption coefficient), yielding temperature distributions and heat flux rates. Among the results of practical interest, there is quantitatively displayed the manner in which the heat transfer decreases as the gas becomes more absorbant.

Heat transfer by conduction through the gas can be included in the formulation. But, its introduction into the problem introduces mathematical non-linearities which significantly complicate the solution of the governing integral equation. In the present study, conduction has not been accounted for. It is interesting to note that the techniques used in setting up the present problem also apply to other geometries.

From a review of the literature on radiation in a parallel plate enclosure, it would appear that previous investigations have given incomplete consideration to the role of the gas. For example, Jakob [1] (p. 105) computes the absorption in a gas layer as radiant energy passes from a hotter to a colder wall. But, the gas is not permitted to re-radiate, as it must under steady state conditions. By ignoring the emission of the gas, Jakob circumvents the problem of computing the temperature distribution of the gas; and as a consequence, his analysis does not permit a complete computation of the net heat transfer. A practical procedure has been proposed by Wohlenberg [1] (p. 132) for computing the net heat transfer in a gas-filled parallel plate enclosure. He ignores the local absorption–emission processes, supposing the gas to be isothermal and to radiate as a whole according to Stephan–Boltzmann's law.

An important engineering approach in the study of non-isothermal radiating gas bodies has been made by Hottel and Cohen [2]. They provide interchange factors between *finite*-sized rectangular gas volumes situated at various orientations with respect to one another. Similar information is also given for surface-to-volume radiation. Such factors are useful when a radiation problem is formulated by subdividing the gas body into a group of finite volumes elements, each of which possesses a uniform temperature different from its neighbors. The approach of the present analysis differs from Hottel and Cohen in that the temperature is permitted to vary continuously throughout the gas and energy conservation is therefore applied to *infinitesimal* volume elements.

Readers who are interested primarily in results are invited to pass over the Analysis section.

ANALYSIS

Conservation of energy

We now proceed to study the purely radiative exchange in the parallel plate system of Fig. 1. Attention is focused on an infinitesimal volume element dV, and the conservation of energy principle is applied. According to this law, the energy content of the element must remain constant in the steady state; and as a consequence, inflow must equal outflow. Energy arrives at the element dV due to radiation emitted at both bounding walls and due to radiation emitted in the remainder of the gas body. In addition, there may be an internal heat source S which also supplies an energy input into dV. For simplicity, it will be supposed that the walls behave as black bodies so that there is no indirect energy transfer due to reflections at the surface. The analysis can be extended to include the effects of non-black walls.

Then, we can write the conservation principle for dV as:

energy absorbed in dV from emission of gas body
+ energy absorbed in dV from emission of lower wall
+ energy absorbed in dV from emission of upper wall
+ internal heat generation
= energy emitted by dV (1)

Our task is to evaluate the various terms of this expression. In the derivation that follows, it will be assumed that the gas occupying the enclosure is gray, so that the absorption coefficient* k is independent of wavelength. Further, k will be taken as independent of temperature. In principle, extension can be made to the situation where k depends on wavelength and/or temperature, with the net result of enormously increasing the difficulty of obtaining solutions to the problem. We will select a spherical element of diameter δ for our infinitesimal volume dV. Separate examination of the terms comprising equation (1) will be made in the ensuing paragraphs.

Energy absorbed in dV from emission of gas body

The derivation of this quantity is facilitated by reference to Fig. 2. We focus attention on the volume dV located at a distance x_0 from the

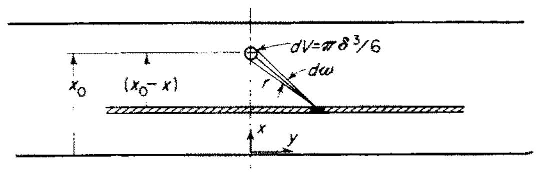

FIG. 2. Diagram for deriving absorption at dV due to gas emission.

lower surface. The first step is to determine the energy absorbed in dV due to the emission of the cross-hatched gas layer situated at a distance $(x_0 - x)$ below dV. Since the temperature depends only on x, the emission per unit volume of such a layer is spatially uniform.

* Jakob uses the alternate designation: logarithmic decrement of radiation.

It will be supposed that the emission from a gas volume $d\tau$ can be written as

$$4ke_g\, d\tau \qquad (2)$$

which includes as a special case the conventional representation [2]

$$4k\sigma T^4\, d\tau \qquad (2a)$$

Under the assumption that an infinitesimal element of the gas radiates uniformly in all directions, it follows that the energy leaving the shaded volume through a solid angle $d\omega$ is

$$4ke_g(dA_\tau\, dx)\,\frac{d\omega}{4\pi} \qquad (3)$$

where $d\tau = dA_\tau\, dx$. Of this amount, there arrives at dV

$$\frac{ke_g}{\pi}\, dA_\tau\, dx\, d\omega\, e^{-kr} \qquad (4)$$

the exponential factor accounting for the absorbing effects of the intervening gas.

Since we are dealing with infinitesimal bodies, the rays arriving at dV from the shaded volume form a *parallel* bundle. As is shown in the Appendix, the energy absorbed (per unit of arriving energy) by a sphere of diameter δ on which there impinges a bundle of parallel rays is

$$\frac{2}{3}\, k\delta \qquad (5)$$

So, of the energy leaving the shaded volume, the amount

$$\frac{2}{3\pi}\, k^2\delta\, e_g\, dA_\tau\, dx\, d\omega\, e^{-kr} \qquad (6)$$

is absorbed in the spherical volume dV.

Now, we proceed to compute the absorption in dV due to energy emitted by the entire cross-hatched strip. From geometrical considerations, making use of symmetry, we have

$$d\omega = \frac{\pi\delta^2/4}{r^2},\; dA_\tau = 2\pi y\, dy,\; r^2 = y^2 + (x - x_0)^2$$

Introducing these relations into (6), we integrate from $y = 0$ to $y = \infty$ and thereby obtain the contribution of the entire strip to the absorption in dV. The result of the integration can be rephrased in the following form

$$2k^2 e_g\, dV \left[\int_{k(x_0 - x)}^{\infty} (e^{-\lambda}/\lambda) d\lambda\right] dx \qquad (7)$$

where the bracketed quantity is called the exponential integral and has been tabulated to high accuracy in reference [3].

The entire gas body can be considered as being made up of a series of cross-hatched strips. Each strip contributes an amount of energy to dV depending on the distance $(x_0 - x)$ and upon the local temperature (i.e. the local value of e_g). The energy absorbed in dV from all the emitting gas strips is found by adding up (integrating) the contributions of each strip. Then, the final expression for the energy absorbed at dV due to the emission of the entire gas body is

$$2k^2\, dV \left\{\int_0^{x_0} e_g \left[\int_{k(x_0-x)}^{\infty} (e^{-\lambda}/\lambda) d\lambda\right] dx + \right.$$
$$\left. + \int_{x_0}^{L} e_g \left[\int_{k(x-x_0)}^{\infty} (e^{-\lambda}/\lambda) d\lambda\right] dx\right\} \qquad (8)$$

Energy absorbed in dV from emission of walls

Consideration is first given to the energy emitted at the lower wall, and attention is directed to Fig. 3. Since it has been assumed that

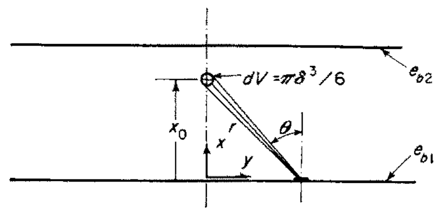

FIG. 3. Diagram for deriving absorption at dV due to surface emission.

the wall is black, the energy leaving the darkened segment of surface area dA_s in the direction θ in a solid angle $d\omega$ is

$$\frac{e_{b1}}{\pi}\, dA_s \cos\theta\, d\omega \qquad (9)$$

This energy travels a distance r before encountering the spherical element dV. As a consequence of absorption in the intervening gas, there arrives at dV

$$\frac{e_{b1}}{\pi} dA_s \cos\theta \, d\omega \, e^{-kr} \tag{10}$$

As already discussed in the preceding section, the sphere (diameter δ) absorbs $2k\delta/3$ of this amount. So, the contribution of the surface area dA_s to dV is

$$\frac{2}{3\pi} \delta \, k e_{b1} \, dA_s \cos\theta \, d\omega \, e^{-kr} \tag{11}$$

Since dA_s is a typical element of the lower wall, the contribution of the entire wall may be found by integrating equation (11). Introducing the relations

$$d\omega = \frac{\pi\delta^2/4}{r^2}, \quad dA_s = 2\pi y \, dy,$$
$$r^2 = y^2 + x^2, \ \cos\theta = x_0/r$$

integration over the range $y = 0$ to $y = \infty$ gives, after rearrangement

$$2k \, e_{b1} \, dV \left[\exp(-kx_0) - \right. \left. - kx_0 \int_{kx_0}^{\infty} (e^{-\lambda}/\lambda) d\lambda \right] \tag{12}$$

Equation (12) represents the energy absorption in dV due to black body emission at the lower surface. In an analogous way, the contribution to dV from black body radiation at the upper surface can be written as

$$2k \, e_{b2} \, dV \left[\exp\{-k(L - x_0)\} - \right. \left. - k(L - x_0) \int_{k(L-x_0)}^{\infty} (e^{-\lambda}/\lambda) d\lambda \right] \tag{13}$$

Internal heat generation

There may be an internal heat source in the gas due to chemical reactions, electric currents, etc. Suppose that the heat generation rate per unit volume is denoted by S; so within dV, there is generated

$$S \, dV \tag{14}$$

Within the framework of our model, S can be a function of x; but it will be taken as a constant here.

Governing equation for the temperature (e_g) *distribution*

We are now in a position to evaluate the conservation equation (1) using the results of the preceding paragraphs. Gathering together the successive terms respectively represented by equations (8), (12), (13) and (14) and equating to the emission as given by equation (2) with $d\tau = dV$, we arrive at

$$\left.\begin{aligned} &\int_0^{x_0} e_g(x) \left[\int_{k(x_0-x)}^{\infty} (e^{-\lambda}/\lambda) d\lambda \right] dx + \\ &+ \int_{x_0}^{L} e_g(x) \left[\int_{k(x-x_0)}^{\infty} (e^{-\lambda}/\lambda) d\lambda \right] dx + \\ &+ e_{b1} \left[\frac{\exp(-kx_0)}{k} - \right. \\ &\qquad \left. - x_0 \int_{kx_0}^{\infty} (e^{-\lambda}/\lambda) d\lambda \right] + \\ &+ e_{b2} \left[\frac{\exp\{-k(L-x_0)\}}{k} - \right. \\ &\qquad \left. - (L - x_0) \int_{k(L-x_0)}^{\infty} (e^{-\lambda}/\lambda) d\lambda \right] + \\ &\qquad + S/2k^2 = \frac{2}{k} e_g(x_0) \end{aligned}\right\} \tag{15}$$

This integral equation governs the variation of the emissive power e_g with x. Since, in general, e_g would be a specific function of temperature, it follows that equation (15) can as be regarded as the governing equation for the temperature distribution.

It is evident that equation (15) is linear in e_g. This suggests a rather convenient reduction of the general problem which includes different wall temperatures ($e_{b1} \neq e_{b2}$) and a non-zero heat source to two simpler problems. One of these is the case of different wall temperatures in the *absence* of a heat source; while the second is the case of an internal heat source with *identical* wall temperatures. The solution of the general case is simply a linear sum of the two. These separate situations are discussed separately below.

Different wall temperatures; no heat source

In the absence of a heat source, it is convenient to introduce the dimensionless emissive power ϕ as follows

$$\phi = \frac{e_g - e_{b1}}{e_{b2} - e_{b1}} \tag{16}$$

Substituting into equation (15) and integrating by parts, we find the following integral equation for ϕ

$$\left.\begin{aligned} &\int_0^{X_0} \phi(X)\left[\int_{kL(X_0-X)}^{\infty} (e^{-\lambda}/\lambda)d\lambda\right] dX + \\ &+ \int_{X_0}^{1} \phi(X)\left[\int_{kL(X-X_0)}^{\infty} (e^{-\lambda}/\lambda)d\lambda\right] dX + \\ &+ \frac{\exp\{-kL(1-X_0)\}}{kL} - \\ &- (1-X_0)\int_{kL(1-X_0)}^{\infty} (e^{-\lambda}/\lambda)d\lambda = \\ &\qquad \frac{2}{kL}\phi(X_0) \end{aligned}\right\} \tag{17}$$

where $X = x/L$. It may be observed that only a single parameter, kL, appears in the integral equation.

It is thus seen that use of the dimensionless emissive power ϕ removes the need to consider specific values for the wall temperature and reduces the problem to a dependence on one dimensionless parameter, kL.

Of practical interest is the calculation of the heat transfer between the walls. In the steady state, the *net* energy transferred from the hotter wall must be identical to the *net* energy transferred to the cooler wall. Focusing attention on the lower wall, we write

net heat transferred to lower wall
= radiation absorbed from emission of gas body
+ radiation absorbed from emission of upper wall − energy emitted (18)

To evaluate the gas radiation term, we first find the contribution from the elementary cross-hatched strip of Fig. 2 and then integrate over all strips. The radiation from the upper wall is evaluated in a straightforward way taking proper account of absorption in the intervening gas. After a lengthy calculation, equation (18) is evaluated to be

$$\left.\begin{aligned} \frac{q}{e_{b2} - e_{b1}} &= 2kL\int_0^1 \phi e^{-kLX} dX - \\ &- 2(kL)^2\int_0^1 \phi X\left[\int_{kLX}^{\infty} (e^{-\lambda}/\lambda)d\lambda\right] dX + \\ &+ e^{-kL}(1-kL) + (kL)^2\int_{kL}^{\infty}(e^{-\lambda}/\lambda)d\lambda \end{aligned}\right\} \tag{19}$$

where q is the net rate of heat transfer per unit area to the lower wall (or from the upper wall). Since $e_{b2} - e_{b1}$ is the net heat transfer in the absence of an absorbing gas, equation (19) immediately gives the fractional reduction in the heat transfer due to the absorbing–emitting medium.

Uniform internal heat source; same wall temperatures

For this situation, we denote the black body emission common to both walls by e_b. Then, a dimensionless emissive power variable is introduced according to the definition

$$\Phi = \frac{e_g - e_b}{S/2k} \tag{20}$$

Under the transformation, it can be shown using integration by parts that equation (15) simplifies to

$$\left.\begin{aligned} &\int_0^{X_0} \Phi(X)\left[\int_{kL(X_0-X)}^{\infty} (e^{-\lambda}/\lambda)d\lambda\right] dX + \\ &+ \int_{X_0}^{1} \Phi(X)\left[\int_{kL(X-X_0)}^{\infty} (e^{-\lambda}/\lambda)d\lambda\right] dX + \\ &\qquad + \frac{1}{kL} = \frac{2}{kL}\Phi(X_0) \end{aligned}\right\} \tag{21}$$

It is interesting to observe that the transformation completely removes the need to consider specific values of the source strength S. The only parameter in the problem is kL. Solution of equation (21) gives the distribution of emissive power (temperature) across the gap, and in particular, the maximum temperature.

The general solution

Since the equations are linear, the general solution can be written in terms of ϕ and Φ. However, care must be taken to dimensionalize them first. Then, the general solution appears in the form

$$e_g = (e_{b2} - e_{b1})\phi + e_{b1} + \frac{S}{2k}\,\Phi + e_b$$

where the lower and upper walls have the emissive powers $(e_b + e_{b1})$ and $(e_b + e_{b2})$, respectively.

RESULTS FOR NO-SOURCE CASE

The governing equation for the no-heat source case, equation (17), has been solved for values of kL equal to 0·1, 0·5, 1 and 2. The solutions were accomplished numerically on a desk calculator using an iterative scheme augmented by the intuition of the operator. Numerical integrations were carried out using the trapezoidal rule and the stability of the solutions was checked by perturbation.

Based on these solutions, the variation of the dimensionless emissive power ϕ across the gap has been plotted in Fig. 4. Utilizing these distributions, the heat transfer has been evaluated

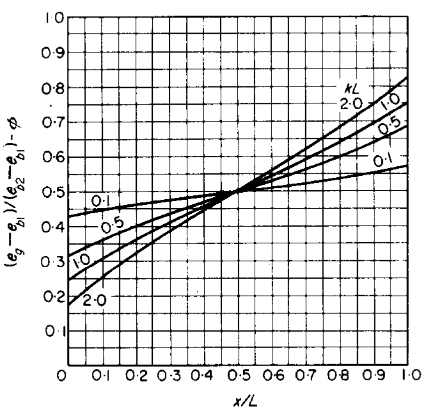

FIG. 4. Distribution of emissive power in a radiating gas between parallel plates (no internal heat source).

from the equation (19) and is shown on Fig. 5. As already noted, the ratio $q/(e_{b2} - e_{b1})$ directly gives the reduction in heat transfer due to the presence of the absorbing–emitting gas.

Turning first to Fig. 4, it may be observed that the curves are relatively flat over a large part of the gap, with the flattening becoming more marked with decreasing values of kL. Consequently, for moderate values of kL, the bulk of the gas body is at a relatively uniform temperature. Under these conditions, the neglect of the temperature dependency of k seems to be vindicated.

The relative uniformity of the temperature in the bulk of the gas suggests that heat conduction effects, which depend on temperature gradients, will be confined to the neighborhood of the walls provided that the thermal conductivity is small. So, there appears to be a boundary layer phenomenon somewhat similar to the situation in fluid dynamics.

The solutions of equation (17), as displayed in Fig. 4, have the property that $e_g \neq e_b$ at the walls ($X = 0$ and $X = 1$). This is completely consistent with our model. In the present problem, where heat conduction is omitted, temperature continuity is not required at the walls. The values taken on by e_g at $X = 0$ and $X = 1$ are simply those required to satisfy energy conservation.

The flattening trend displayed on Fig. 4 with decreasing kL has its analogue in the heat conduction problem in rarefied gases. In that instance, the temperature distribution across the gap passes through a sequence of shapes with decreasing gas density which are qualitatively similar to the curves of Fig. 4. The two phenomena have a qualitatively similar explanation. In the rarefied conducting gas, a decrease in density leads to an increase in mean free path for molecular collisions. For the absorbing–emitting gas, a decrease in kL leads to an increase

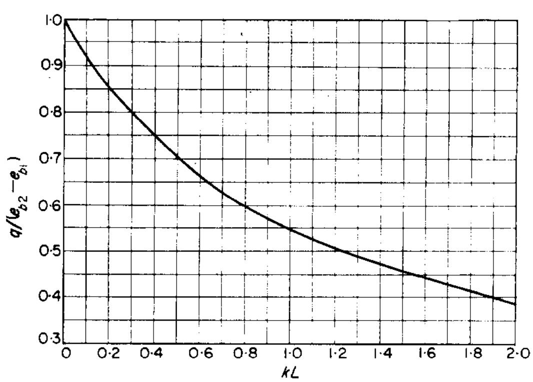

FIG. 5. Heat transfer between parallel plates separated by a radiating gas (no internal heat source).

in the mean free path (relative to the gap spacing) for photon collisions.

Next, we turn to Fig. 5. The finding that the heat transfer q decreases with increasing kL is quite expected. For instance, it is physically reasonable that increasing the gap spacing L for a gas of given k must increase the resistance to radiative heat flow. A similar conclusion applies when the absorption coefficient is increased for a gap of fixed spacing L. The results of Fig. 5 should be useful in choosing an "insulating gas" to diminish the radiative heat exchange between two surfaces.

RESULTS FOR THE HEAT SOURCE CASE

Solutions of the integral equation (21) which governs the uniform internal heat source case have been carried out numerically for values of kL equal to 0·1, 0·5, 1 and 2. The distribution of the dimensionless emissive power Φ obtained from these solutions is plotted in Fig. 6. In common with the findings of the previous section, it is seen that the curves are relatively flat over a major portion of the gap. The tendency towards flatness increases with decreasing kL. So, for moderate kL there is again an essentially isothermal zone over the major portion of the gas, and the neglect of the temperature dependence of k is vindicated.

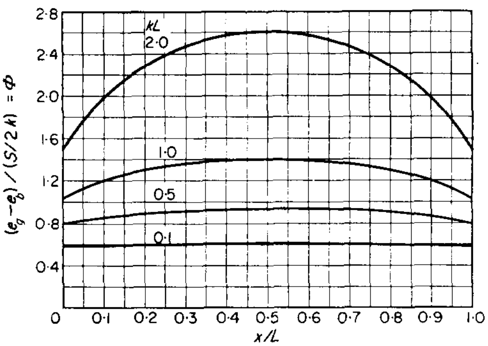

FIG. 6. Distribution of emissive power in a heat generating and radiating gas between parallel plates.

It is interesting to study the separate effects of the various physical parameters L, k, and S on the energy (temperature) distribution. Fig. 6 is well suited to study the effect of changing the gap spacing L. For fixed values of k, S and e_b, the only parameter which changes from curve to curve is L. As expected, the temperature level increases as the spacing increases. For small gap spacings, the gas body is essentially isothermal.

Next, we proceed to display the effect of changing k while holding S, L and e_b fixed. Fig. 6 is inadequate, since k appears both as a parameter on the curves and as part of the ordinate variable. To illustrate the effect, we consider the special case where $S/2ke_b = 1$ when $kL = 1$. (Then, as k changes and the other quantities are held fixed, $S/2ke_b$ must change in a corresponding manner.) Fig. 7 has been constructed

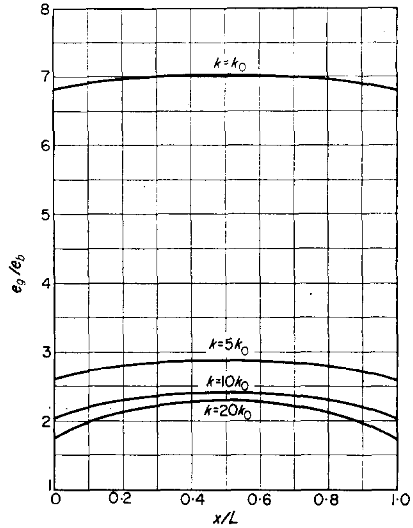

FIG. 7. Effect of k on distribution of emissive power in a heat generating gas for fixed S, L and e_b and $k_0 = S/20e_b$, $L = 2e_b/S$.

on this basis, the distribution of e_g being plotted against x/L with k decreasing by a factor of 20 from the lowest to the highest curve. From the figure, we note the interesting result that the temperature level *increases* as the absorption coefficient k decreases and that this effect is more accentuated at lower values of k. This suggests that the gas temperature may be minimized by using a gas with a high absorption coefficient.

Finally, we can show the effect of changing the source strength S while holding k, L and e_b fixed. For purposes of illustration, consideration is given to the situation where $kL = 1$. Fig. 8,

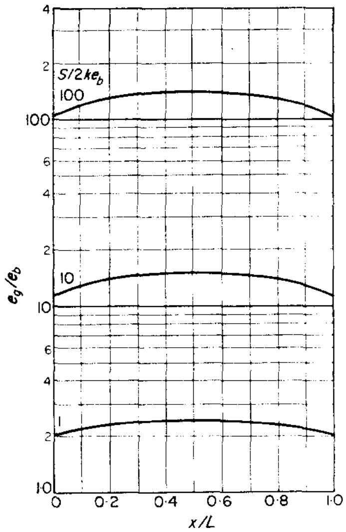

FIG. 8. Effect of heat source S on distribution of emissive power for fixed k, L, and e_b and $kL = 1$.

drawn for values of $S/2ke_b$ of 1, 10 and 100 shows the variation of e_g with distance across of gap. The level of the curves is seen to increase linearly with the source strength, as might have been expected from the linearity of the problem.

CONCLUDING REMARKS

Within the framework of the simplifying assumptions, it has been possible to give an exact formulation for the problem of radiation between two plane walls separated by a non-isothermal absorbing–emitting gas. Numerical solutions have shown the quantitative response of the temperature distribution and heat transfer rate to changes in such parameters as absorption coefficient k, gap spacing L, and heat source S.

For moderate values of kL, the temperature of the gas body is nearly uniform over a large part of the gap, in some measure justifying the neglect of the temperature dependence of k.

Further work involving a more complicated model is clearly indicated. Heat conduction effects would appear to be important in the region near the wall. A more refined treatment might well account for the temperature dependence of k and might consider the non-gray characteristics of real gases. Finally, a variable heat source might also be included.

APPENDIX

Absorption of Parallel Radiation by a Sphere

According to a Nusselt's derivation (reproduced in [1], pp. 100–101), the energy absorbed by a sphere of diameter δ from a parallel bundle of rays of intensity J is

$$J\left\{1 + \frac{2}{(k\delta)^2}\left[[e^{-k\delta}(1 + k\delta) - 1\right]\right\} \quad \text{(A1)}$$

Now, we wish to find the limit of equation (A1) for an infinitesimal sphere. First, the exponential term is expanded in series

$$e^{-k\delta} = 1 - k\delta + \frac{(k\delta)^2}{2} - \frac{(k\delta)^3}{6} + \ldots$$

and then introduced into (A1). After cancellation and retaining only first order infinitesimals, we find that the absorbed energy is

$$J\left(\frac{2}{3}\right)k\delta \quad \text{(A2)}$$

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